Benha University
Faculty Of Engineering at Shoubra

ECE 122
Electrical Circuits (2)(2016/2017)
Lecture (5)
Filters

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Filters

A **filter** is a circuit that is designed to pass signals with desired frequencies and reject or attenuate others.

- The filter can be treated as a networks designed to have frequency selective behavior
- A filter can be used to limit the frequency spectrum of a signal to some specified band of frequencies.
- Filters are the circuits used in radio and TV receivers to allow us to select one desired signal out of a multitude of broadcast signals in the environment

. **Filters** may be classified as either digital or analog.

. **Digital filters** are implemented using a digital computer or special purpose digital hardware.

. **Analog filters** may be classified as either passive or active and are usually implemented with R, L, and C components and operational amplifiers.

Filters

Background:

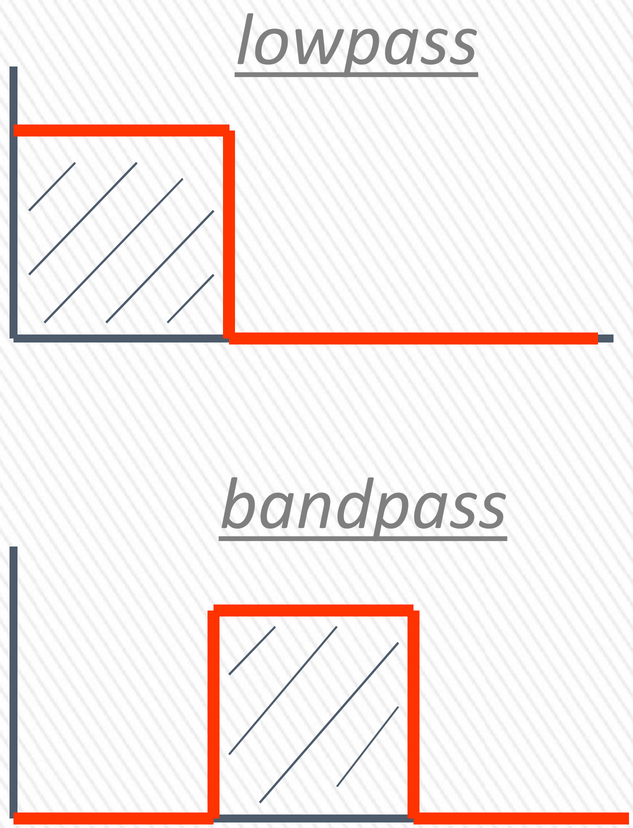
- An *active filter* is one that, along with R, L, and C components, also contains an energy source, such as that derived from an operational amplifier.

- A *passive filter* is one that contains only R, L, and C components. It is not necessary that all three be present. L is often omitted (on purpose) from passive filter design because of the size and cost of inductors – and they also carry along an R that must be included in the design.

Passive Analog Filters

Background:

Four types of filters - "Ideal"

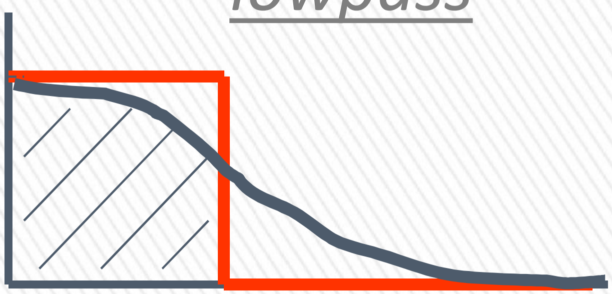


Passive Analog Filters

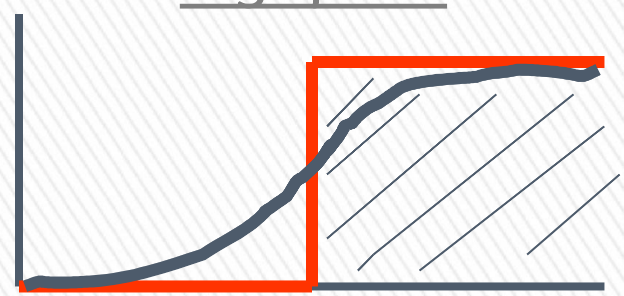
Background:

Realistic Filters:

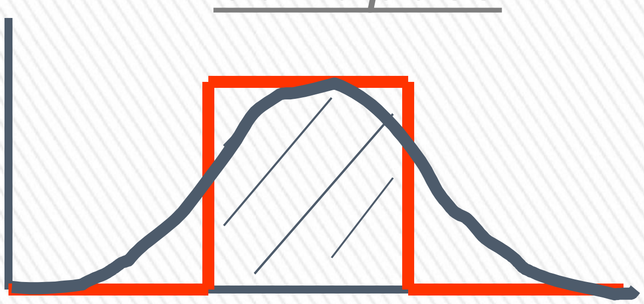
lowpass



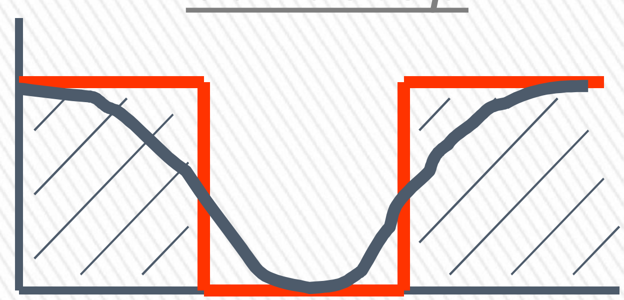
highpass



bandpass



bandstop



Filters

Important Table for determining the type of the filter from its Transfer function

Summary of the characteristics of ideal filters.

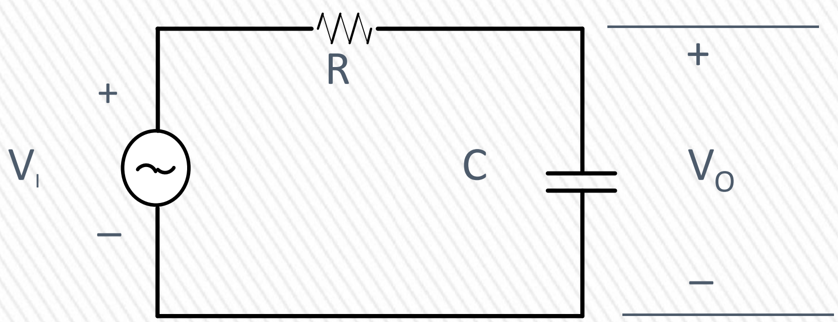
Type of Filter	$H(0)$	$H(\infty)$	$H(\omega_c)$ or $H(\omega_0)$
Lowpass	1	0	$1/\sqrt{2}$
Highpass	0	1	$1/\sqrt{2}$
Bandpass	0	0	1
Bandstop	1	1	0

Low Pass Filter

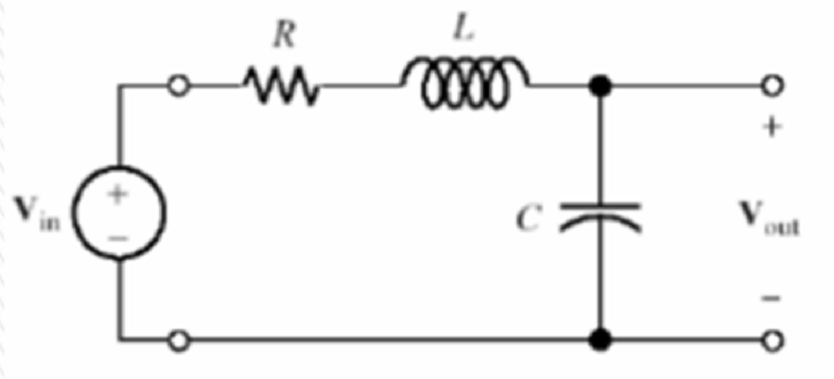
Passive Analog Filters

Low Pass Filter

Consider the circuit below.



Low pass filter circuit

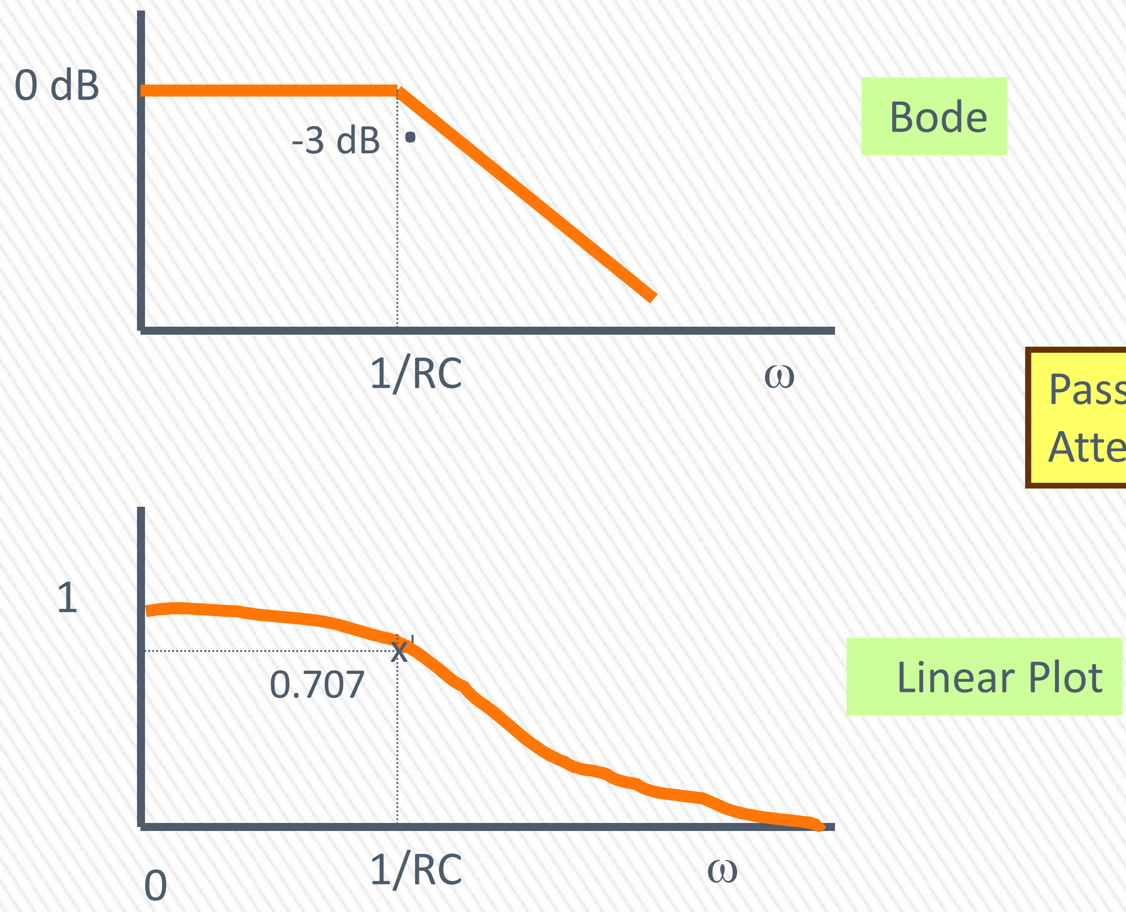


$$\frac{V_O(jw)}{V_i(jw)} = \frac{\frac{1}{jwC}}{R+\frac{1}{jwC}} = \frac{1}{1+jwRC}$$

- At low frequency the capacitor is an open circuit (Vo = Vin)
- At high frequency the capacitor is a short and the inductor is open (Vo = 0)

Passive Analog Filters

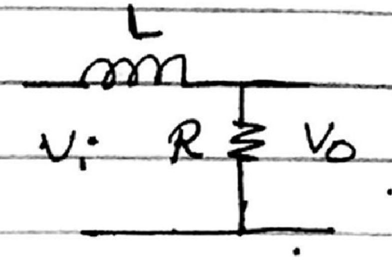
Low Pass Filter



Passes low frequencies
Attenuates high frequencies

Example (1)

Show that a series LR circuit is a low-pass filter if the output is taken across the resistor. Calculate the corner frequency fc if L= 2 mH and R= 10 kΩ.



$$V_o = V_i \frac{R}{R + j\omega L}$$

$$\frac{V_o}{V_i} = H(f) = \frac{R}{R + j\omega L}$$

at $\omega = 0 \rightarrow H(f) = 1$

$\omega = \infty \rightarrow H(f) = 0$

(LPF)

Corner Freq. at $|H(f)| = \frac{1}{\sqrt{2}} H(0)$

$$H(f) = \frac{R}{R + j\omega L} = \frac{1}{1 + j\frac{\omega L}{R}}$$

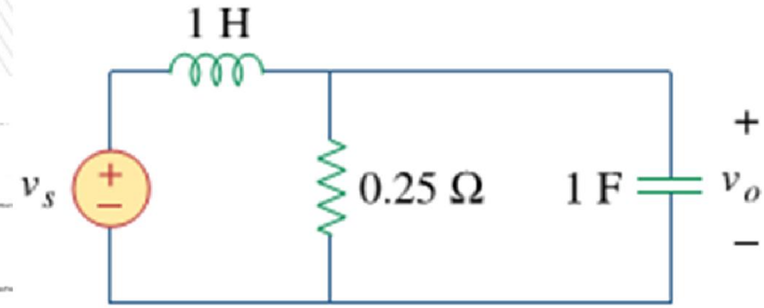
$$|H(f)| = \frac{1\angle 0}{\sqrt{1 + (\frac{\omega L}{R})^2}\angle\theta} = \frac{1}{\sqrt{1 + (\frac{\omega L}{R})^2}} = \frac{1}{\sqrt{2}}$$

$\therefore 1 + (\frac{\omega L}{R})^2 = 2$ or $\frac{\omega L}{R} = 1$ $\therefore \omega = \frac{R}{L}$

$\therefore 2\pi f = \frac{R}{L}$ $\therefore f = \frac{1}{2\pi}\frac{R}{L} = 796\,kHz$

Example (2)

Find the transfer function Vo/Vs of the circuit in Figure. Show that the circuit is a low-pass filter.



$$H(f) = \frac{R // x_C}{x_L + R // x_C}$$

$$= \frac{R\left(\frac{1}{j\omega C}\right) / \left(R + \frac{1}{j\omega C}\right)}{j\omega L + \dfrac{R\left(\frac{1}{j\omega C}\right)}{R + \frac{1}{j\omega C}}}$$

$$= \frac{R\left(\frac{1}{j\omega C}\right)}{\left(R + \frac{1}{j\omega C}\right)(j\omega L) + R\left(\frac{1}{j\omega C}\right)}$$

$$= \frac{R}{R + (j\omega C)(j\omega L)\left(R + \frac{1}{j\omega C}\right)}$$

Example (2)

$$= \frac{R}{R - \omega^2 LC\left(R + \frac{1}{j\omega C}\right)}$$

$$= \frac{R}{R - \omega^2 LRC - \frac{1}{j}\omega L} = \frac{R}{R - \omega^2 LRC + j\omega L}$$

at $\omega = 0 \rightarrow H(f) = 1$

$\omega = \infty \rightarrow H(f) = 0$

$\Rightarrow$ LPF

Example (3)

Determine the cutoff frequency of the low-pass filter described by

$$\mathbf{H}(\omega) = \frac{4}{2 + j\omega 10}$$

Find the gain in dB and phase of H (ω) at ω= 2 rad/s.

Sol/ cutoff Freq calculated when

$$|H(\omega)| = \frac{1}{\sqrt{2}} |H(\omega)|_{max}$$

$$H(0) = \frac{4}{2} = 2 \rightarrow \therefore max$$

$$H(\infty) = 0$$

$\therefore$ cutoff Freq calculated when $|H(\omega)| = \frac{1}{\sqrt{2}}(2)$

But
$$|H(\omega)| = \frac{4\angle 0}{\sqrt{2^2 + (10\omega)^2}\angle\theta} = \frac{\cancel{4}\,2}{\sqrt{4 + 100\omega^2}} = \frac{\cancel{2}\,1}{\sqrt{2}}$$

Example (3)

$$\therefore \frac{4}{4+100w^2} = \frac{1}{2} \qquad \therefore 100w^2 + 4 = 8$$

$$\text{or } w^2 = \frac{4}{100} \qquad \boxed{w = 0.2} \text{ rad/t}$$

$$H(2) = \frac{4}{2+j20} = \frac{2}{1+j10}$$

$$|H(2)| = \frac{2\angle 0}{\sqrt{101}\ \angle \tan^{-1} 10} = \frac{2\angle 0}{\sqrt{101}\ \angle 84.3^\circ}$$

$$\text{Phase } (0 - \tan^{-1} 10) = -84.3^\circ$$

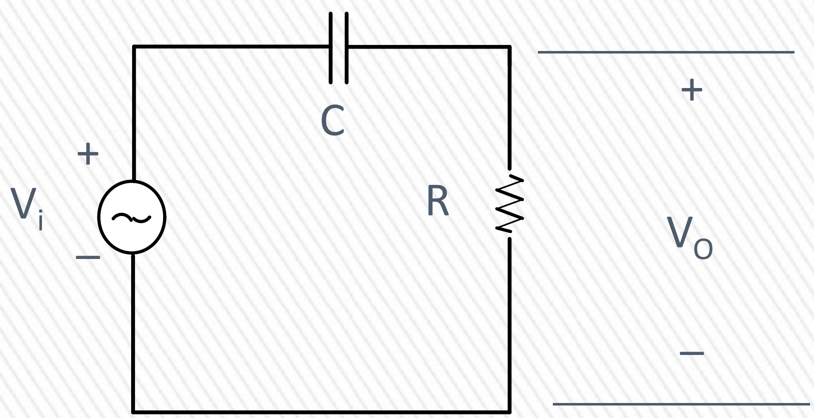
$$\text{Mag} = \frac{2}{\sqrt{101}} = 0.199$$

$$\text{Mag}_{dB} = 20 \log 0.199 = -14.023$$

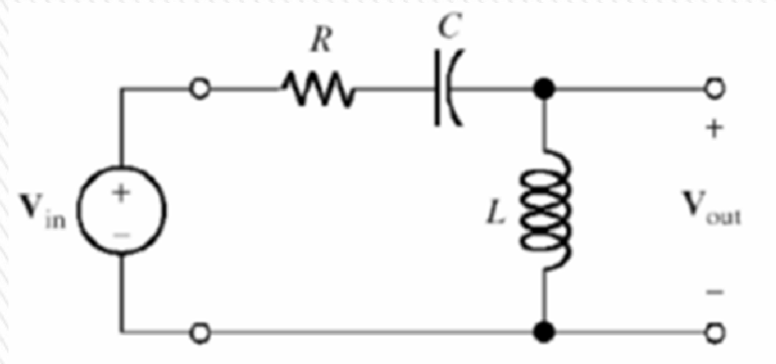
High Pass Filter

Passive Analog Filters

High Pass Filter



High Pass Filter



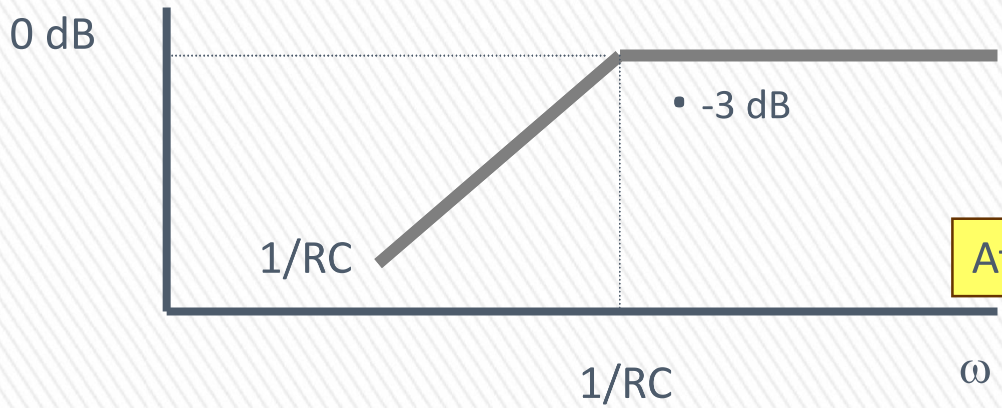
$$\frac{V_o(jw)}{V_i(jw)} = \frac{R}{R+\frac{1}{jwC}} = \frac{jwRC}{1+jwRC}$$

- At low frequency the capacitor is an open circuit (Vo = 0)
- At high frequency the capacitor is a short and the inductor is open (Vo = Vin)

Passive Analog Filters

High Pass Filter

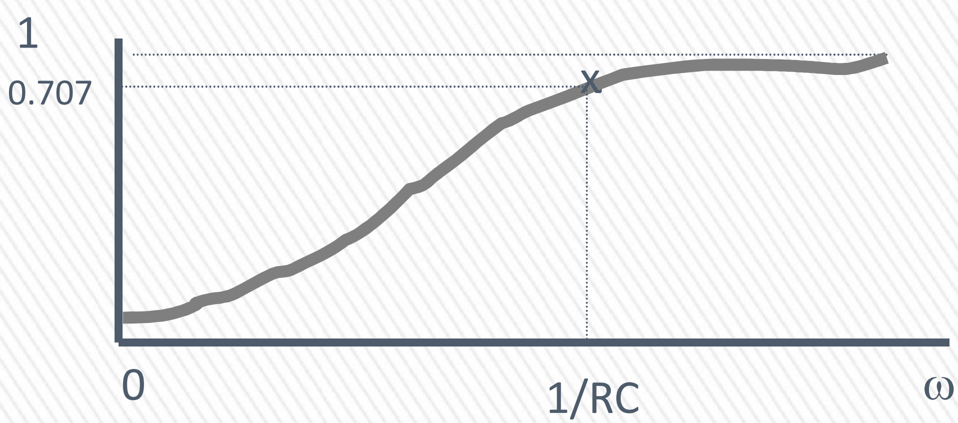
Bode



Passes high frequencies

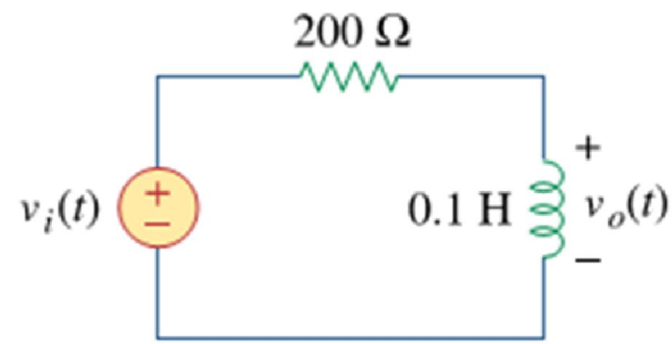
Attenuates low frequencies

Linear



Example (4)

Determine what type of filter in figure . Calculate the corner frequency fc



Sol/

$$V_o = V_i \left(\frac{X_L}{R + jX_L} \right)$$

$$H(f) = \frac{j\omega L}{R + j\omega L} = \frac{1}{1 + \frac{R}{j\omega L}}$$

at $\omega = 0 \rightarrow H(f) = 0$

$\omega = \infty \rightarrow H(f) = 1$

$\therefore$ HPF

$$\rightarrow |H(\omega)| = \frac{1}{\sqrt{2}} \max = \frac{1}{\sqrt{2}} H(\infty) = \frac{1}{\sqrt{2}}$$

$$= \left| \frac{1}{1 + \frac{R}{j\omega L}} \right| = \left| \frac{1}{1 - j\frac{R}{\omega L}} \right| = \frac{1}{\sqrt{1 + \left(\frac{R}{\omega L}\right)^2}} = \frac{1}{\sqrt{2}}$$

or $1 + \left(\frac{R}{\omega L}\right)^2 = 2$ $\quad \therefore \frac{R}{\omega L} = 1$

$\therefore \omega = R/L$, $2\pi f = R/L \rightarrow f = 318.3$ Hz

Example (5)

In a high-pass RL filter with a cutoff frequency of 100 kHz, L= 40 mH. Find R.

Sol// So it is RL

$$\omega = \frac{R}{L} = 2\pi f$$

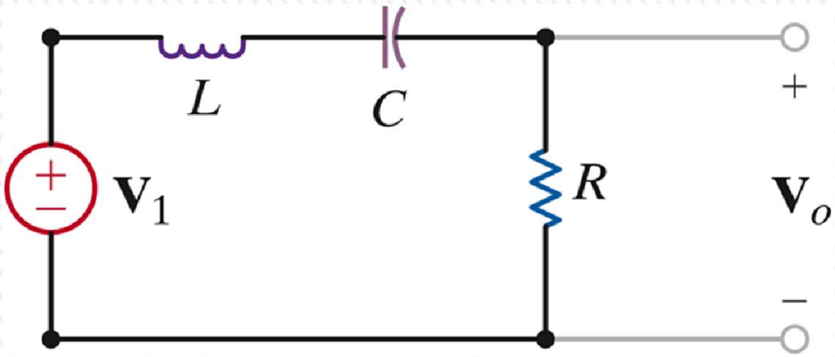
$L \rightarrow 40mH$, $f \rightarrow 100k$

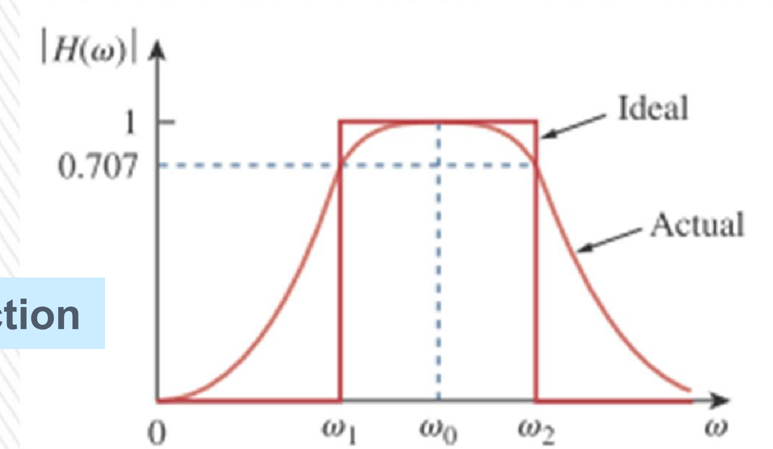
$$R = 25.13 k\Omega$$

Band Pass Filter

Simple Passive band-pass filter

The RLC series resonant circuit provides a bandpass filter when the output is taken off the resistor as shown in Fig.





$$H = \frac{V_0}{V_1} = \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

Transfer Function

$$|H(\omega)| = \frac{\omega RC}{\sqrt{(\omega RC)^2 + (\omega^2 LC - 1)^2}}$$

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$H\left(\omega = \frac{1}{\sqrt{LC}}\right) = 1$$

$$H(\omega=0) = H(\omega=\infty) = 0$$

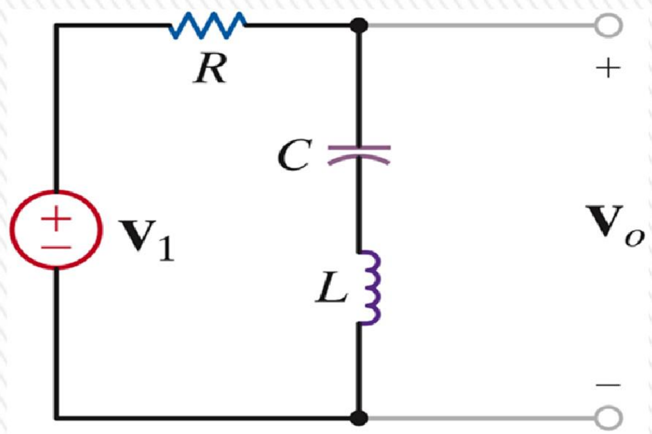
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$H(\omega_{LO}) = \frac{1}{\sqrt{2}} = M(\omega_{HI})$$

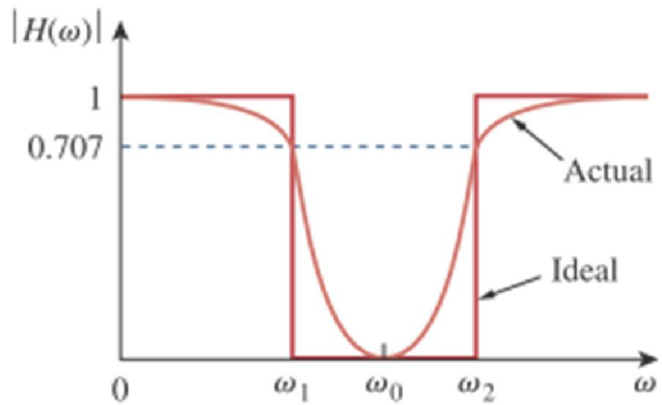
$$BW = \omega_{HI} - \omega_{LO} = \frac{R}{L}$$

Band Stop Filter (reject) (notch)

Simple Passive band-stop filter



- The RLC series resonant circuit provides a band-stop filter when the output is taken off the LC as shown in Fig.
- A filter that prevents a band of frequencies between two designated values
- It is also known as a band-stop, band-reject, or notch filter.



$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j(\omega L - 1/\omega C)}{R + j(\omega L - 1/\omega C)}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow j\left(\omega_0 L - \frac{1}{\omega_0 C}\right) = 0$$

at $\omega = 0$ the capacitor acts as open circuit $\Rightarrow V_0 = V_1$

at $\omega = \infty$ the inductor acts as open circuit $\Rightarrow V_0 = V_1$

BW = Bandwidth of rejection

ω_1, ω_2 are determined as in the band - pass filter

$$\mathbf{H}(0) = 1, \ \mathbf{H}(\infty) = 1.$$

Active Filters

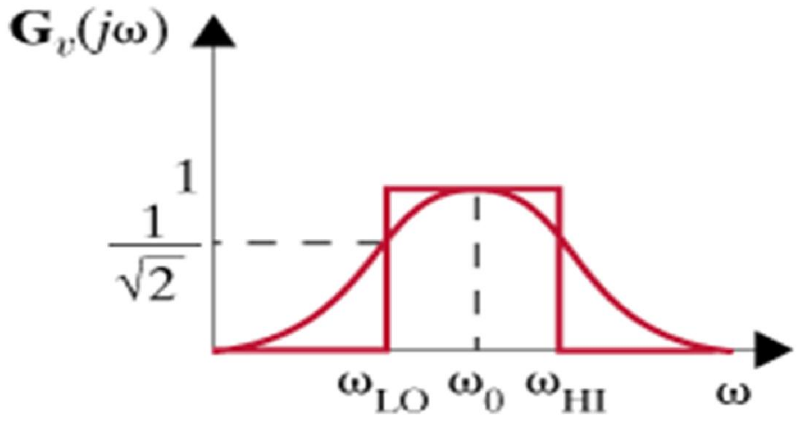
Passive filters have several limitations:

1. Cannot generate gains greater than one
2. Loading effect makes them difficult to interconnect
3. Use of inductance makes them difficult to handle

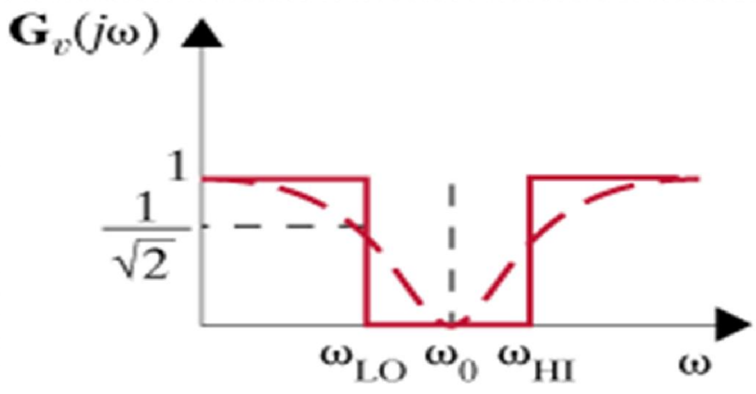
- Using operational amplifiers one can design all basic filters, and more, with only resistors and capacitors

Remember

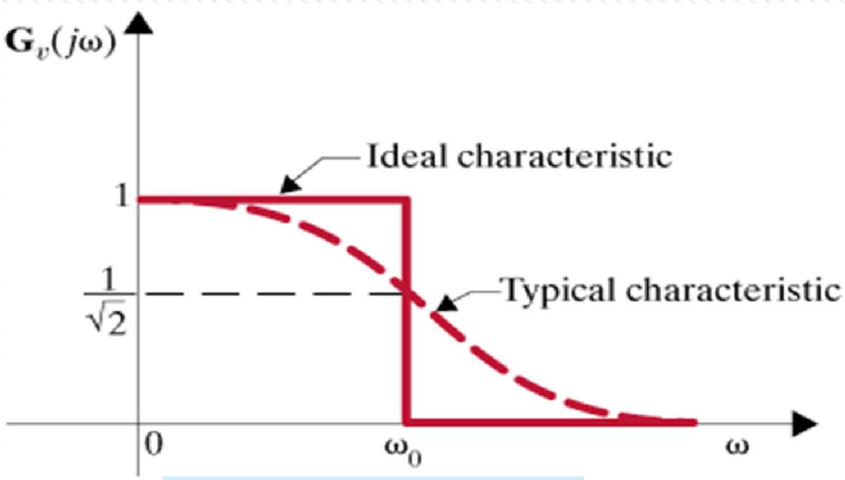
Filter types Remember



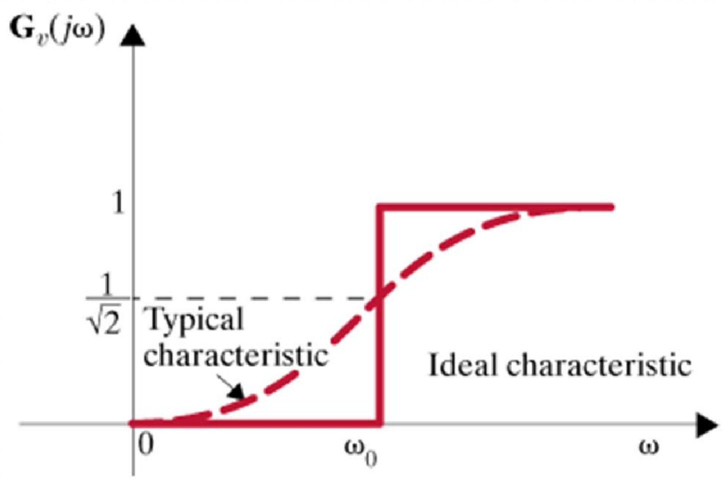
Band-pass filter



Band-reject filter



Low-pass filter



High-pass filter

Thank You